

Comment on “Particle Path Through a Nested Mach-Zehnder Interferometer”

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(Dated: August 9, 2016)

In a recent paper, arXiv:1604.04596, Griffiths questioned—based on an informative consistent-histories (CH) argument—the counterfactuality, for one of the bit choices, of Salih et al.’s protocol for communicating without sending physical particles, Phys. Rev. Lett. 110 (2013) 170502. Here, we first show that for the Mach-Zehnder version used to explain our protocol, no family of consistent histories exists where any history has the photon travelling through the communication channel, thus rendering the question of whether the photon was in the communication channel meaningless from a CH viewpoint. We then show that for the actual Michelson-type protocol, there are consistent-histories families that include histories where the photon travels through the communication channel. We show that the probability of finding the photon in the communication channel is zero—thus confirming complete counterfactuality.

PACS numbers: 03.67.Dd, 03.67.Hk, 03.65.Ta

The protocol in question for direct counterfactual quantum communication is first explained in reference [1] using nested Mach-Zehnder interferometers, much easier to explain, before the actual protocol is given in Michelson form. The Michelson implementation captures the key functionality of its Mach-Zehnder counterpart while allowing a substantial saving of physical resources. But, as we explain below, not only is the Michelson implementation more powerful in this sense, it also allows counterfactuality to be verified in a way that is not possible using Mach-Zehnder interferometry.

The debate about counterfactuality for the bit value corresponding to Bob not blocking the communication channel can be explained using only two nested Mach-Zehnder interferometers. (Counterfactuality is not questioned for the bit value corresponding to Bob blocking the channel.) We reproduce the key figure from [2] in our FIG. 1 below. In order to frame the debate in the context of counterfactual communication, imagine the two nested interferometers of FIG. 1 rotated by 45 degrees clockwise with the interferometers and detectors thought of as being in Alice’s station on the left—except arm C, which would correspond to the communication channel leading to Bob on the right.

By constructing a family of consistent histories that includes a history where the photon takes path A between S at time t_0 , and F at time t_4 , we can ask what the probability of the photon taking path A is. As shown in [2], such a family exists, namely, $\mathcal{F}'_A : S_0 \odot \{A_1, D_1, Q_1\} \odot \{A_2, B_2 + C_2\} \odot \{A_3, E_3, H_3\} \odot F_4$. Using the extended Born rule, the probability of the photon taking path A is one, meaning that the photon remains in Alice’s domain at all times. What about the probability of finding the photon in the communication channel, C . In general, as shown in [2], there is no consistent histories family that includes a history where the photon travels through C ,

rendering the question meaningless—except for a special choice of reflectivity for mirrors 1 and 4. This reflectivity, equal to $1/3$, however, lies outside the parameter space of Salih et al.’s protocol which has reflectivity $\cos^2 \frac{\pi}{2N}$, with N ranging from 2 for the smallest number of cycles, giving reflectivity $1/2$, to asymptotically large in the ideal case. In fact a reflectivity of $1/3$ would correspond to a probability of correctly guessing Bob’s bit choice (when Bob does not block the channel) equal to $1/3$, which is worse than random guessing. In other words there is no consistent histories family with the photon travelling through the communication channel.

However, this is not the only reason why such a family does not exist. The basis of the counterfactuality of Salih et al.’s protocol, as stated in [3], is that “the probability of the photon existing at location E is zero”, which the analysis in section 7 in [2] does not explicitly take account of. In particular, Griffiths’s proposition $P1$, “The particle was in S at t_0 and in F at t_4 ” does not include “the particle was not in E at t_3 ”. Thus \mathcal{F}_C has to be refined to include events at time t_3 , which, as shown in [2], will make it inconsistent regardless of mirror reflectivity choice. Whereas one can say based on CH that the photon remains in channel A in Alice’s domain at all times, one cannot say that the photon was in the communications channel at any time.

We now give, for the Michelson-type protocol, whose inner working is explained in detail in the caption of FIG. 2, a consistent-histories family that includes a history where the photon travels through the communication channel C . The Michelson version allows us to ask what the probability of finding the photon in the communication channel is shortly after time t_3 . The consistent histories family is $S_0 \odot \{A_1, D_1\} \odot \{A_2, B_2 + C_2\} \odot \{A_3, B_3, C_3\} \odot F_4$. The corresponding chain kets are mutually orthogonal since they are all, with the exception of $|S_0, A_1, A_2, A_3, F_4\rangle$, zero. It is easy to see that $|S_0, D_1, B_2 + C_2, B_3, F_4\rangle$ is zero because after two applications of SPR_2 in the inner interferometer, the polarisation of an entering V photon would have been rotated

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all the way to H , meaning nothing would be reflected by PBS_2 into B at time t_3 . (This is equivalent in FIG. 1 of complete destructive interference at E .) It is also easy to see from FIG. 2 that $|S_0, D_1, B_2 + C_2, C_3, F_4\rangle$ is zero because if the photon is found in C shortly after time t_3 it would inevitably end up at detector D_3 . Therefore, by the extended Born rule, the probability of finding the photon in the communication channel, $\langle S_0, D_1, B_2 + C_2, C_3, F_4 | S_0, D_1, B_2 + C_2, C_3, F_4 \rangle$, is zero—confirming counterfactuality. And because all chain kets other than $|S_0, A_1, A_2, A_3, F_4\rangle$ are zero, the probability that the photon has remained in Alice's domain at all times is one. The same applies if we have two or more outer cycles.

What about the probability of finding the photon in the communication channel shortly after time t_2 ? Consider the histories family, $S_0 \odot \{A_1, D_1\} \odot \{A_2, B_2, C_2\} \odot \{A_3, B_3, C_3\} \odot F_4$. The corresponding chain kets for individual histories are all mutually orthogonal since they are all zero except the chain ket $|S_0, A_1, A_2, A_3, F_4\rangle$. This is easy to see because for a photon exiting the inner interferometer, polarising beamsplitter PBS_1 would pass any H component towards detector D_3 while reflecting any V component towards PBS_0 , which would in turn direct it towards D_2 and, crucially, away from D_1 . This family of histories is therefore consistent. By the extended Born rule, the probability of finding the photon in the communication channel C at any time is zero. What happens if we have two (the minimum for our protocol) or more outer cycles? We first note that subsequent outer cycles work the same way as the first one. Consider the case of two outer cycles, with SPR_1 and SPR_2 applying the same rotations as in FIG. 1. For the first outer cycle, however, the histories family $S_0 \odot \{A_1, D_1\} \odot \{A_2, B_2, C_2\} \odot \{A_3, B_3, C_3\} \odot F_4$ would be inconsistent, rendering the question of whether the photon was in the communication channel meaningless.

The reason is that subsequent polarisation rotation at the start of the second outer cycle, by SPR_1 , would make the chain ket $|S_0, D_1, C_2, B_3, F_4\rangle$ nonzero (meaning complete destructive interference at time t_3 is no longer fulfilled). This would also make the less refined family $S_0 \odot \{A_1, D_1\} \odot \{A_2 + B_2, C_2\} \odot \{B_3, A_3 + C_3\} \odot F_4$ inconsistent. What about the question of whether the photon was in the communication channel during the second outer cycle. Because this is the last outer cycle, for a photon exiting the inner interferometer, polarising beamsplitter PBS_1 would pass any H component towards detector D_3 while reflecting any V component towards PBS_0 , which would in turn direct it towards D_2 and away from D_1 . The corresponding histories family is now consistent. The probability of finding the photon in the communication channel is zero. More generally, for any number of cycles, we can ask what the probability of finding the photon in the communication channel is during the last outer cycle. The answer is always zero. And, again, because all chain kets other than the one corresponding to the history where the photon remains in A are zero, the probability that the photon has remained in Alice's domain at all times is one.

In summary, based on consistent histories, the photon was in Alice's domain at all times. Moreover, whenever the question of whether the photon was in the communication channel is deemed meaningful from a consistent histories viewpoint, the answer is always no—confirming complete counterfactuality.

ACKNOWLEDGMENTS

Qubet Research is a start-up in quantum communication technology.

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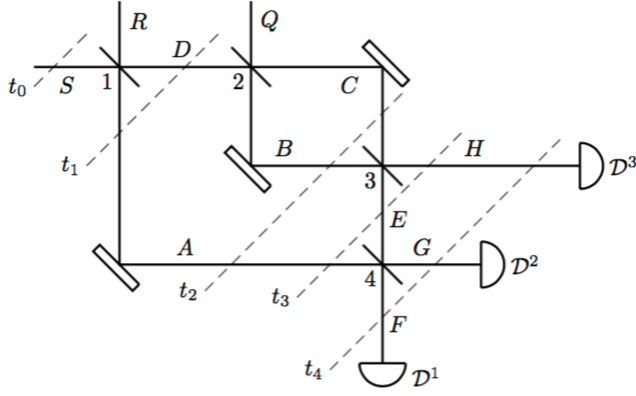


FIG. 1. This diagram, reproduced from [2], captures the essence of the Mach-Zehnder counterfactual communication protocol of Salih et al.. One has to picture the two nested interferometers rotated by 45 degrees clockwise, with the whole setup thought of as belonging to Alice, except path C which would correspond to the communication channel, and the mirror therein (double lines) which would belong to Bob.

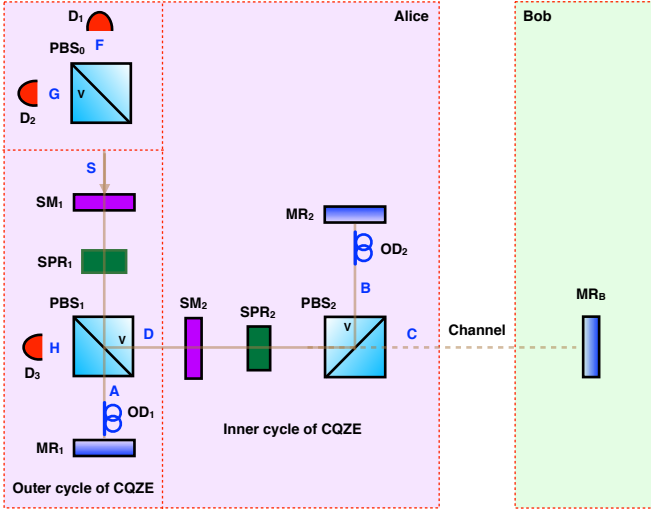


FIG. 2. The inner working of the Chained Quantum Zeno Effect (CQZE) for the case of Bob not blocking the channel. In order to illustrate the operation of the CQZE in a way that mirrors the setting in FIG. 1, we use only two outer and two inner CQZE cycles. The letters in blue correspond to those denoting the different channels in FIG. 1. Let us start with Alice's photon in S at time t_0 . (We have omitted the photon source from the diagram in order to simplify it.) Switchable mirror SM_1 is switched off letting Alice's H photon in before it is switched on again. Using switchable polarisation rotator SPR_1 the following rotation is applied to the photon, $|H\rangle \rightarrow 1/\sqrt{2}(|H\rangle + |V\rangle)$, before it is switched off. At time t_1 , the V part of the superposition is reflected (through D) towards Bob using polarising beamsplitter PBS_1 , while the H part is passed (through A) towards MR_1 . Switchable mirror SM_2 is then switched off to let the V part of the superposition into the inner interferometer before it is switched on again. Using switchable polarisation rotator SPR_2 , the following rotation $|V\rangle \rightarrow 1/\sqrt{2}(|V\rangle - |H\rangle)$ is then applied before it is switched off for the rest of this inner cycle. At time t_2 , polarising beamsplitter PBS_2 passes the H part of the superposition (through C) towards Bob while reflecting the V part (through B) towards MR_2 . With Bob not blocking the channel, the H and V parts inside the inner interferometer are reflected back by MR_B and MR_2 before being recombined by PBS_2 . During the second inner cycle, the polarisation of the part of photon superposition inside the inner interferometer is rotated by SPR_2 all the way to $-H$ before being passed by PBS_2 at time t_3 (through C) towards MR_B . (Note that the fact that nothing is reflected by PBS_2 towards SM_2 , through B , is equivalent in FIG. 1 of complete destructive interference at E .) Switchable mirror SM_2 is then switched off to allow this part of the superposition out. Measurement by D_3 leaves the photon in the overall state $|H\rangle$ moving towards SM_1 , unless it is lost to D_3 . (Note that for the case of two or more outer cycles, subsequent outer cycles would be identical.) Switchable mirror SM_1 is then switched off to allow the photon, whose final state is $|H\rangle$, out. At time t_4 , polarising beamsplitter PBS_0 allows the photon towards detector D_1 (through F). Optical delays OD ensure that effective path-lengths match. Note that unlike in [1], but similar to [4, 5], $SPR_{1(2)}$ is applied once at the beginning of each outer(inner) cycle, instead of being applied twice, each time effecting half the desired rotation. This is key to our analysis, and eliminates an error, negligible for a large number of cycles, caused otherwise by an undesired polarisation rotation by SPR_1 of the exiting photon.